LASER
PRINCIPLES

Chapter Three

Resonator

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CHAPTER THREE: RESONATOR

3-1) Resonator Definition

It is a system consisting of two mirrors on a common optical axis, where laser photons move back and forth between the two mirrors for amplification. It is the main source of feedback, as it works when the active material is placed as a suitable resonant oscillator, forming what are called standing waves. The resonator is also called the optical cavity, and it is an essential element because it:

- Supports the amplification in the active medium resulting from stimulated emission (responsible for feedback).
- Directs the beam (giving the laser a high directivity).
- Maintains the longitudinal and transverse oscillation mode of the laser emission.

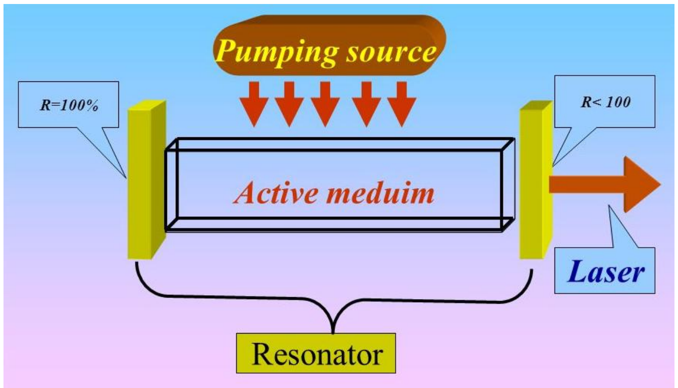

Figure (3-1): A schematic diagram showing the basic components of a laser device.

The important factor in laser production is the mirrors installed on both sides of the laser production material. The reflectivity of the rear mirror is (100%), while the transmittance of the front mirror ranges between (1% - 5%). The rear mirrors are usually made of aluminum, silver, or copper. The coating material is chosen based on the wavelength of the laser. The mirrors help reflect some photons back into the laser material several times. These photons stimulate other excited electrons to release more photons of the same wavelength and the same phase. This is the process of light amplification, as the resonator works to amplify the amount of light radiation per unit time, which leads to increased gain and thus laser generation.

The reflectivity $\check{R}$ of metal mirrors is calculated based on the laser wavelength λ and ρ represents the density of the metal, as follows:

$$\check{R} = 100 - 3.65\sqrt{\frac{\rho}{\lambda}} \ldots\ldots\ldots\ldots \text{(3-1)}$$

For a laser system to function properly, optical alignment must be maintained. This involves positioning the two mirrors in a perfectly aligned position so that the optical axis is straight. This alignment depends on the type of active medium, the size of the system, and the degree of complexity of the system.

3-2) Resonator Types

There are several types of resonators depending on the nature of the application in which the laser is used. The main types of resonators are as follows:

1- Plane Parallel Mirrors Resonator:

It is called a Fabry-Perot resonator. This type of resonator is characterized by the large size of the laser beam emerging from it, low damage to the mirrors, high efficiency in exciting the active medium, and difficulty in controlling the type of the emerging transverse pattern. The values of the radii of the two mirrors are ($R_1 = R_2 = \infty$).

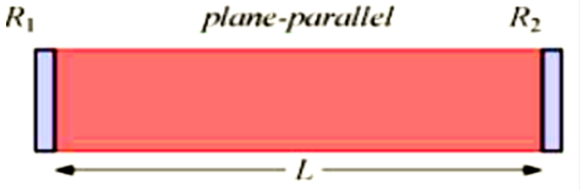

Figure (3-2): Parallel plane mirror resonator.

2- Confocal Mirrors Resonator:

In this type, the focus of the first mirror is coincident with the focus of the second mirror, and the size of the laser beam becomes smaller under the following condition: ($R_1 = R_2 = L \rightarrow R_1 + R_2 = 2L$). This type is characterized by the ease of optical alignment, and the loss of high transverse (non-Gaussian) modes is large, and the Gaussian mode is about 25 times larger than the rest of the modes of the same laser beam.

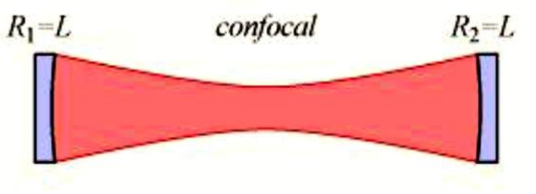

Figure (3-3): Confocal mirrors resonator.

3- Concentric Resonator:

This resonator consists of two mirrors whose sum of the radii of curvature is equal to the length of the distance between them ($R_1 + R_2 = L$). If ($R_1 = R_2 = R \rightarrow L = 2R$), then this resonator is called a spherical resonator. This type is characterized by the difficulty of optical alignment and the fact that the transverse pattern emerging from it is large.

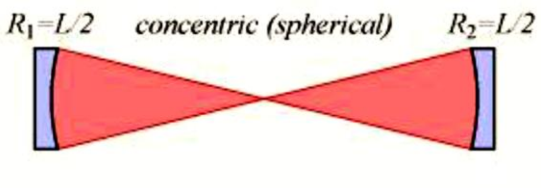

Figure (3-4): Concentric resonator.

4- Large-Radii Resonator:

In this type, the radii of curvature of the mirrors are much larger than the length of the resonator. This type is characterized by the ease of optical alignment and the fact that the smallest radius of the laser beam is at the waist region.

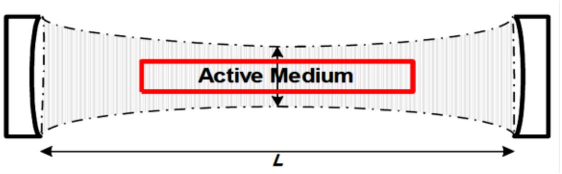

Figure (3-5): Large-radii resonator.

5- Hemispherical Resonator:

This type consists of a plane mirror and a concave mirror, so the size of the transverse pattern is as large as possible for the concave mirror and as small as possible for the plane mirror. The values of the radii of the two mirrors are ($R_1 = L$, $R_2 = \infty$).

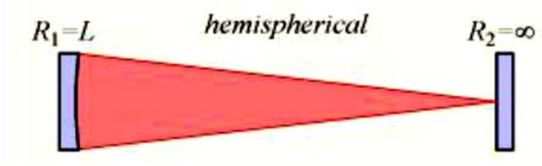

Figure (3-6): Hemispherical resonator.

6- Unstable Resonator:

This resonator consists of two mirrors, one concave and the other convex, and is used at high power levels in the laser system.

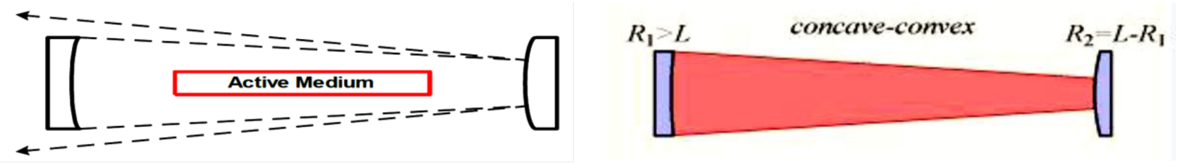

Figure (3-7): Unstable resonator.

In general, the resulting resonator shapes can be classified into two types: the stable resonator, which is the resonator designed so that its mirrors confine the beam inside the laser cavity and oscillate back and forth towards the optical axis, preventing it from exiting; and the unstable resonator, in which the beam deviates after repeated reflections in a direction away from the resonator axis.

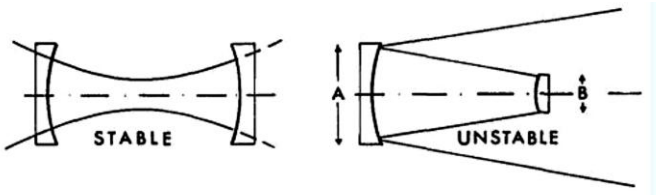

Figure (3-8): Stable and unstable resonators.

3-3) Resonator Stability

A spherical resonator generally contains two spherical mirrors. The radius of curvature of the mirror is positive if the mirror is concave and negative if it is convex. This affects the number of oscillation modes, frequency and amplitude. Thus, the resonator may be stable or unstable. In the case of a stable resonator, the concavity of the two mirrors is such that the light is kept centered near the axis of the resonator. However, in the case of an unstable resonator, the light rays continue their movement away from the axis of the resonator, resulting in a loss.

The condition for the stability of the resonator is:

$$0 \leq g_1 g_1 \leq 1 \text{ ………… } (3\text{-}2)$$

$$g_1 = 1 - \frac{L}{R_1} \text{ ………… } (3\text{-}3)$$

$$g_2 = 1 - \frac{L}{R_2} \text{ ………… } (3\text{-}4)$$

The stability condition of the resonator can be represented by a diagram known as the stability diagram in which g_2 is a function of g_1 and in which the two boundary curves are defined when ($g_1 g_2 = 1$). The shaded areas satisfy the stability condition (they are subject to a stable resonator) and the unshaded areas are subject to an unstable resonator, i.e. they do not satisfy the stability condition. The dotted straight line that makes an angle of (45°) with the axes g_1 and g_2 represents resonators whose mirrors each have two equal radii.

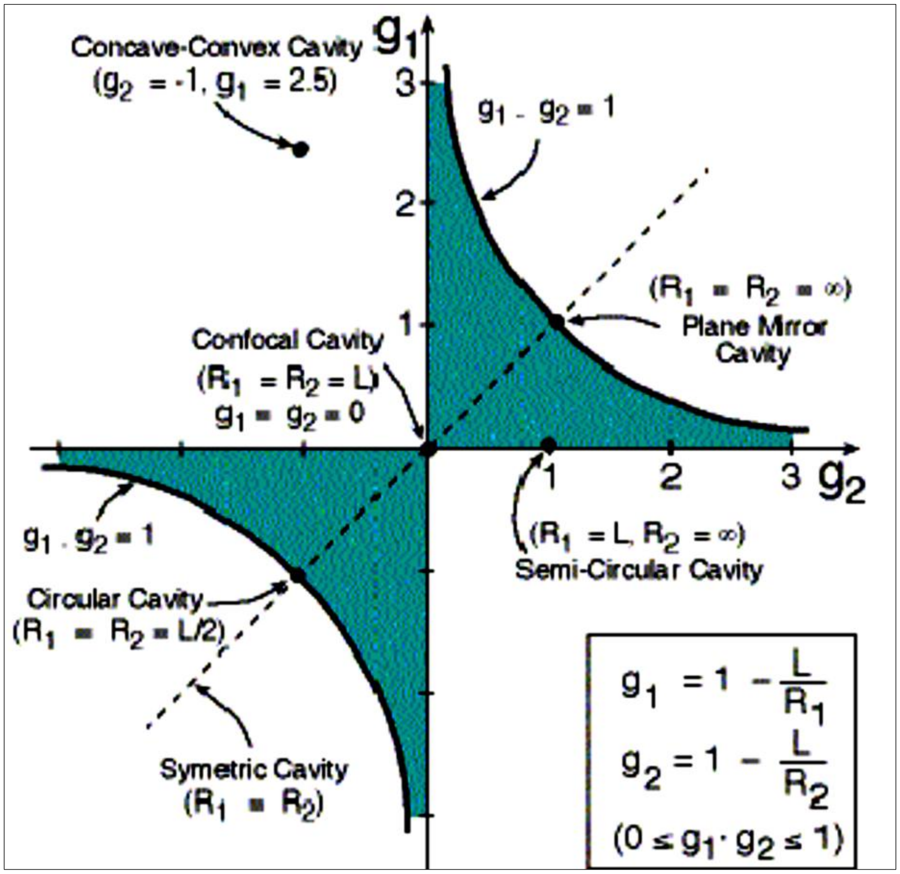

Figure (3-9): Stability diagram of the resonator.

❖ **Eg.:** Determine the stability of a spherical resonator with a radius of curvature of its mirrors R if the distance between them L is equal to: (R/2), (R), (2R).

Sol.:

$g_1g_2 = (1 - \frac{L}{R_1})(1 - \frac{L}{R_2})$, $0 \leq g_1g_1 \leq 1$

1) $g_1g_2 = \left(1 - \frac{2R}{R}\right)\left(1 - \frac{2R}{R}\right) = (1-2)(1-2) = 1$ Stable (metastable)

2) $g_1g_2 = \left(1 - \frac{R}{R}\right)\left(1 - \frac{R}{R}\right) = (1-1)(1-1) = 0$ Stable (metastable)

3) $g_1g_2 = \left(1 - \frac{R}{2R}\right)\left(1 - \frac{R}{2R}\right) = (1-0.5)(1-0.5) = 0.25$ Stable

❖ **Eg.:** Calculate the stability of the following resonators:

1) L=1.5 m, R_1=3 m, R_2=2 m
2) L=1 m, R_1=0.5 m, R_2=2 m
3) L=1 m, R_1=3 m, R_2= -2 m

Sol.:

1) $g_1g_2 = \left(1 - \frac{1.5}{3}\right)\left(1 - \frac{1.5}{2}\right) = (1-0.5)(1-0.75) = 0.125$ Unstable

2) $g_1g_2 = \left(1 - \frac{1}{0.5}\right)\left(1 - \frac{1}{2}\right) = (1-2)(1-0.5) = -0.5$ Unstable

3) $g_1g_2 = \left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{-2}\right) = \left(\frac{2}{3}\right)\left(\frac{3}{2}\right) = 1$ Stable (metastable)

3-4) Emission Linewidth

Laser beams are characterized by being mono-wavelength, meaning that their frequency bandwidth (Δν) is very narrow. Theoretically, the emission linewidth is equal to a small fraction of a hertz (Hz). However, scientifically, the emission linewidth of a laser beam expands greatly for many reasons. Therefore, the laser beam contains a large number of frequencies. In this case, the laser beam is called multimode.

The emission linewidth (Δv) of a laser beam is defined as the response of the atoms of the active medium to the electromagnetic field. It is the set of wavelengths or frequencies generated as a result of stimulated emission and laser generation. It is the lowest possible theoretically (equal to 1) in the case of a laser, but in practice it is slightly wider. There are two reasons for expanding the emission linewidth:

1) **Homogeneous broadening:** Its causes are both collision broadening and natural broadening.
2) **Inhomogeneous broadening:** Its causes are both impurity ions in crystals and Doppler broadening.

3-5) Resonator Paterns

The resonator patterns are expressed by resonator oscillation formulas, which represent the shape and distribution of the electromagnetic field intensity at any position inside and outside the resonator and depend on the shape and dimensions of the two mirrors and the distance between them.

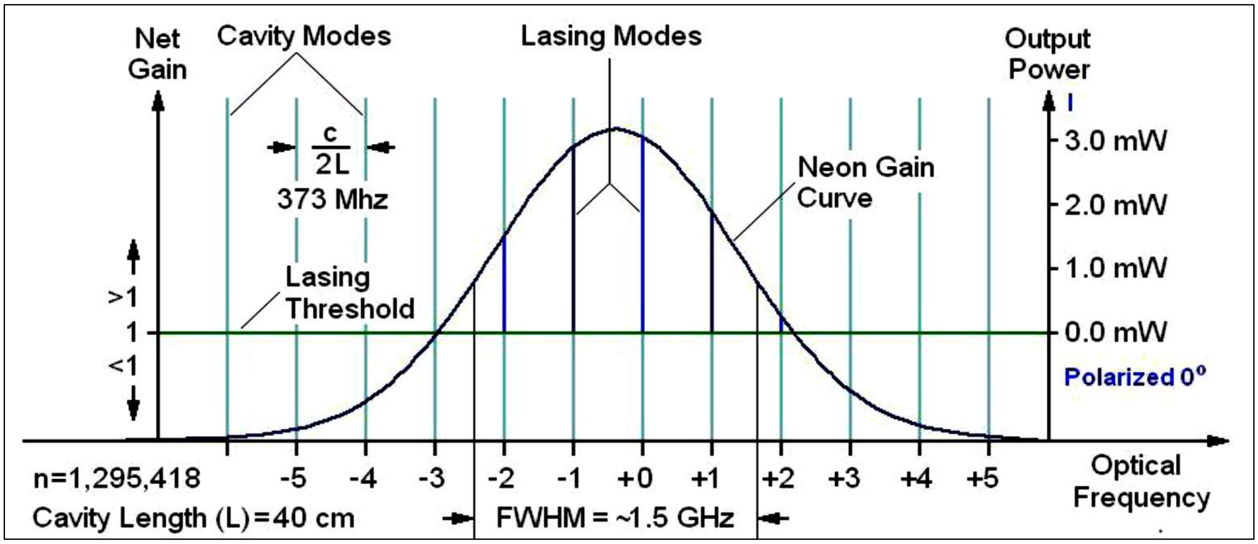

Figure (3-10): Longitudinal oscillation formulas within the laser emission curve.

Feedback is used to produce lasers using mirrors to amplify the light beam as it passes through the laser's radiating medium. These mirrors play a role in influencing the electromagnetic radiation inside the amplifier, producing two types of patterns: longitudinal patterns and transverse patterns.

1) **Longitudinal Modes:** They are also called axial modes. They are the distributions of the electromagnetic field of the laser wave parallel to the optical axis of the resonator. The reason for the formation of these modes is due to the formation of standing waves between the two mirrors. They are formed as a result of the superposition of two coherent waves that propagate in opposite directions. The distance between two successive longitudinal modes (which is called the frequency interval) is given by the equation:

$$\delta = \frac{c}{2L} \quad \text{………… (3-5)}$$

Longitudinal patterns are calculated from the following equation:

$$N_m = \frac{2L}{\lambda} = \frac{2L}{c}\Delta\nu \quad \text{………… (3-6)}$$

The number of longitudinal oscillation modes depends on the spectral linewidth and the length of the resonator. The longer the resonator, the smaller the frequency separation between two successive modes. This leads to a greater number of oscillation modes within the laser emission line. Measures taken to control the number of longitudinal modes of the laser beam include:

- Cooling the active medium.
- Controlling the length of the resonator.
- Using a quality factor control technique.

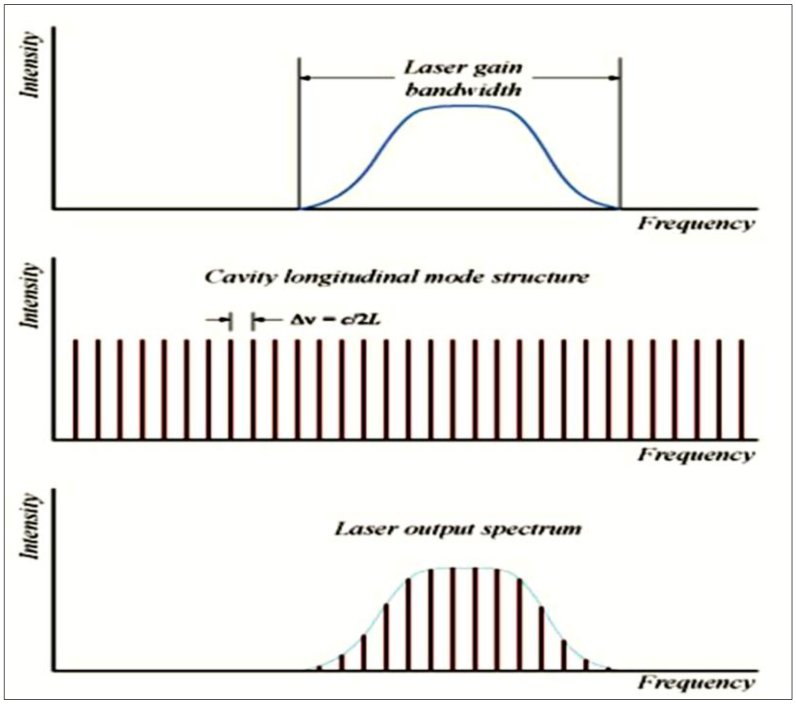

Figure (3-11): Laser emission longitudinal oscillation formulas.

2) **Transvers Modes:** It is the shape of the distribution of the intensity of the electromagnetic field perpendicular to the direction of the optical axis. By studying the distribution of the intensity of laser rays over the cross-sectional area perpendicular to the optical axis of the laser, it was found that it takes different shapes depending on the accuracy of the mirror location, and that any slight change leads to a change in these shapes, which are known as transverse patterns. By projecting a laser beam onto a white screen after magnifying it with a diverging lens, the transverse patterns of the laser beam can be examined.

The number of transverse oscillation modes depends on the shape and size of the mirror. When a number of them are present in the laser output, the laser is said to be multi-mode. Transverse Electromagnetic Modes (TEM) describe the shape of the energy distribution in the cross-section of the beam. Each transverse mode (TEM) is characterized by two coefficients, so TEM_{mn}, where (m, n) are integers, assuming the beam advances in the direction Z (*m* represents the number of zero-light points (between bright areas) along the X-axis, *n* represents the number of zero-light points (between bright areas) along the Y-axis).

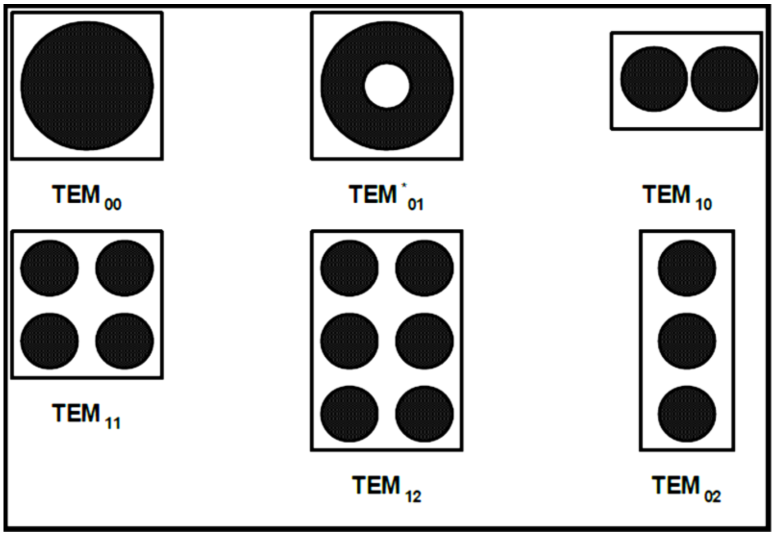

Figure (3-12): Transverse electromagnetic patterns.

The TEM_{00} mode is called the fundamental mode or Gaussian mode. This mode contains about (85 %) of the energy of the resulting laser beam. It is circular in shape (regular) and is considered the most preferred mode in operating laser systems in general. The TEM^*_{01} mode is called the cake mode. This mode is produced due to the presence of impurities or particles on the surface of the mirror or inside the active medium. The higher-order mode is more divergent than the lower-order mode.

3-6) Optical Feedback

A laser works like any other electronic oscillator. An oscillator is a device that produces vibrations without an external influence, such as a microphone or speaker, when they operate in a closed circuit, with an amplifier between them. The laser oscillator works on the same principle, where a portion of the amplified photons are returned by the stimulated emission process using a resonator to be amplified and obtain the characteristic of directionality.

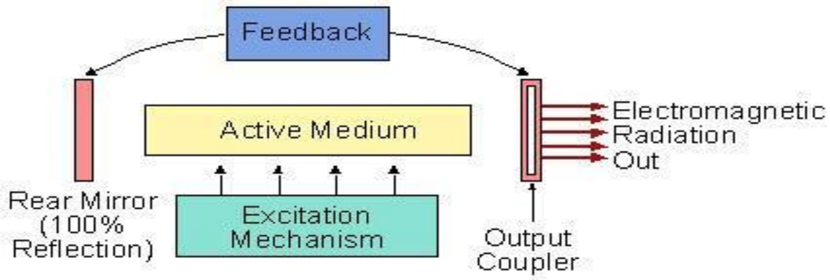

Figure (3-13): Optical feedback.

When photons of intensity I_o fall through the laser amplifier material, they are magnified by an amount G and the intensity of the rays becomes I_oG. Using a mirror R_2, part of the rays is reflected by an amount R_2 and the intensity of the rays becomes I_oGR_2. The mirror returns the rays to the amplifier again so that the rays are magnified by an amount G again and exit I_oGR_2G to fall on the other mirror R_1. The intensity of the rays upon reflection is $I_oGR_2GR_1$. This is what happens to the rays when they enter the amplifier during one magnification cycle. The magnification gained is in the amount GG and the loss in the rays is due to R_1R_2. In other words, the basic condition for the oscillator to work as a signal amplifier is that the final result after one cycle is greater than the original signal I_o, so ($I_oGR_2GR_1 \geq I_o$).

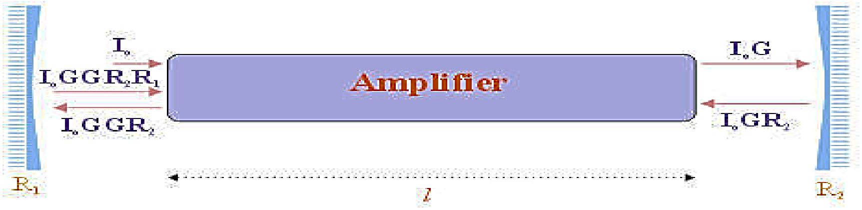

Figure (3-14): Amplification of a single cycle in a resonator.

Considering that not all of the generated photons exit the active medium, but only a part of them, a term called gain was coined, which is the ratio of the energy consumed in stimulation (electrical, optical, etc.) to the energy exiting the active medium in the form of a laser beam.

3-7) Reverse Population and Threshold Condition

To achieve high gain in stimulated emission, it is necessary to reduce all causes of loss in the laser device. One of these causes is absorption by the resonator mirrors. To reduce such losses, dielectric coatings with high absorption capacity are used to coat the mirrors with multiple layers instead of metal coatings. These layers are deposited sequentially on the base material (glass). Due to the phase difference at the point of contact between any two layers, all reflected rays are in phase and interfere constructively. Typically, more than twenty layers are used to achieve a reflectivity close to (99.9 %). For lower reflectivity, the number of layers is reduced. The total loss in the laser device is due to various factors. Although their amounts vary depending on the type of laser, most of them appear to be common, the most important of which are:

- **Losses in the active medium:** As a result of the medium absorbing a wide range of pumping energy, transitions occur that are unrelated to the laser transmission, in addition to losses resulting from scattering due to loss of optical homogeneity (as in solid-state lasers).
- **Transmittance in resonator mirrors:** The transmittance of one of the mirrors is essential because it represents the laser output window, in addition to absorption, scattering, and diffraction losses.

The gain coefficient expresses the increase in optical intensity, and its value is related to the difference between the number of atoms in the excited state and their number in the normal state, in addition to other variables of the medium.

Let us assume that γ represents the loss that reduces the gain coefficient G to $(G - \gamma)$. To calculate the change in radiation *intensity* resulting from one rotation of the radiation inside the resonator (back and forth), we assume a medium that fills the gap between the two mirrors (M_1, M_2), which have a reflective capacity (R_1, R_2) respectively, and are at a distance (L) between them. Then:

$$I = I_\circ e^{-\alpha L} \ldots\ldots\ldots\ldots (3\text{-}7)$$

Where I and I_o represent the intensity of the transmitted and incident radiation respectively, and α represents the absorption coefficient or the material's attenuation coefficient for the radiation and is equal to $(-(G - \gamma))$.

After reflection from mirror M_2, the intensity will become:

$I = R_2 I_\circ e^{(G-\gamma)L}$ ……….. (3-8)

After a full cycle it will be:

$I = R_1 R_2 I_\circ e^{(G-\gamma)2L}$ ……….. (3-9)

Therefore, the amount of gain after a complete cycle (Γ) is expressed as the ratio between the final intensity and the initial intensity, i.e.:

$\Gamma = \frac{I}{I_\circ} = R_1 R_2 e^{(G-\gamma)2L}$ ……….. (3-10)

- If the gain is greater than one, the beam oscillates and grows (we get amplification).
- If the gain is less than one, the oscillation does not continue and fades away, and we do not get amplification (there is no laser beam).
- If the gain is equal to one, it is called the threshold gain factor. Therefore, the threshold condition can be written as follows:

$R_1 R_2 e^{(G_{th}-\gamma)2L} = 1$ ……….. (3-11)

$G_{th} = \gamma + \frac{1}{2L} ln \frac{1}{R_1 R_2}$ ……….. (3-12)

For a continuous wave laser, the threshold gain coefficient (G_{th}) is the same as the steady state gain, γ represents the loss in the active medium, and the second term represents the loss in the resonator design, which includes the useful leakage of the laser output. In terms of inverse qualification,**:**

$G = \sigma(N_2 - N_1)$ ……….. (3-13)

$\sigma = \frac{Bnh\nu g_{(\nu)}}{c}$ ……….. (3-14)

Where g(v) represents the spectral line shape function and at the threshold the qualification has a critical value:

$N_c = (N_2 - N_1)_c = \frac{G_{th} c}{B_{21} nh\nu g_{(\nu)}}$ ……….. (3-15)

3-8) Quality Factor of Resonator

The quality factor of a resonator is defined as the ability of the resonator to store electromagnetic energy within it and represents the ratio between the stored energy and the dissipated energy during one cycle:

$$\text{Quality Factor} = \frac{2\pi \times Stored\ Energy}{Dissipated\ Energy}$$

A resonator has a high quality factor when it stores energy well, while a resonator with a low quality factor does not. In addition, a high quality factor is accompanied by a narrow spectral line, and a low quality factor is accompanied by a relatively wide spectral line. The mathematical formula for the quality factor is represented by the ratio between the frequency of the laser beam (ν) and the width of the emission line (Δν), i.e.:

$$Q = \frac{\nu}{\Delta\nu} = \frac{4\pi\nu L}{c(1-R_1R_2)} \ldots\ldots\ldots\ldots \text{(3-16)}$$

The quality factor also depends on the resonant frequency ($\nu_\circ$), the time rate of power lost ($\acute{P}$) per cycle (round trip of the laser beam) and the cycle time (T).

$$Q = \frac{2\pi\nu_\circ T}{\acute{P}} \ldots\ldots\ldots\ldots \text{(3-17)}$$

❖ **Eg.:** Calculate the quality factor of a laser system with a wavelength of (632.8 nm) and a front mirror with a reflectivity of (95%) and a resonator length of (1 m), then calculate the width of the emission line (Δν).

Sol.:

$R_1 = 100\ \%$

$$Q = \frac{\nu}{\Delta\nu} = \frac{4\pi\nu L}{c(1-R_1R_2)}$$

$$Q = \frac{4\pi\times1}{632.8\times10^{-9}(1-1\times0.95)} = 3.99 \times 10^8$$

$$Q = \frac{\nu}{\Delta\nu} \rightarrow \Delta\nu = \frac{\nu}{Q} = \frac{\frac{c}{\lambda}}{Q} = \frac{\frac{3\times10^8}{632.8\times10^{-9}}}{3.99\times10^8} = 1.18\ MHz$$