Determine the number of quantum states N in Silicon between E_c and $E_c + 0.05\ \mathrm{eV}$ at $T = 300\ \mathrm{K}$.

Given:

- Effective mass of electrons: $m_n^* = 1.08m_0$
- Planck's constant: $h = 6.626 \times 10^{-34}\ \mathrm{J}$\cdotps
- Thermal energy: $kT = 0.02585\ \mathrm{eV}$ (converted to Joules)

Solution:

1. **Convert Thermal Energy kT to Joules:**

$$kT = 0.02585\ \mathrm{eV} \times 1.602 \times 10^{-19}\ \mathrm{J/eV} = 4.14 \times 10^{-21}\ \mathrm{J}$$

2. **Density of States Function $g_c(E)$:**

   The density of states $g_c(E)$ for conduction band electrons is given by:

$$g_c(E \geq E_c) = \frac{4\pi(2m_n^*)^{3/2}}{h^3}\sqrt{E - E_c}$$

3. **Integral to Find Number of States N:**

   To find the number of states between E_c and $E_c + 0.05\ \mathrm{eV}$, integrate $g_c(E)$ from E_c to $E_c + 0.05\ \mathrm{eV}$:

$$N = \int_{E_c}^{E_c+0.05\ \mathrm{eV}} g_c(E)\, dE$$

4. **Substitute the Expression for $g_c(E)$:**

$$N = \int_{E_c}^{E_c+0.05\ \mathrm{eV}} \frac{4\pi(2m_n^*)^{3/2}}{h^3}\sqrt{E - E_c}\, dE$$

5. **Change of Variables:**

   Let $x = E - E_c$, so $dE = dx$. The limits of integration become 0 to 0.05 eV:

$$N = \frac{4\pi(2m_n^*)^{3/2}}{h^3} \int_0^{0.05\ \mathrm{eV}} \sqrt{x}\, dx$$

6. **Evaluate the Integral:**

   The integral of $\sqrt{x}$ with respect to x is:

$$\int_0^{0.05\ \mathrm{eV}} \sqrt{x}\, dx = \frac{2}{3} x^{3/2} \Big|_0^{0.05\ \mathrm{eV}}$$

$$\int_0^{0.05\ \mathrm{eV}} \sqrt{x}\, dx = \frac{2}{3}(0.05)^{3/2}$$

$$(0.05)^{3/2} = 0.00052\ \mathrm{eV}^{3/2}$$

$$\int_0^{0.05\ \mathrm{eV}} \sqrt{x}\, dx \approx \frac{2}{3} \times 0.00052 = 0.00034\ \mathrm{eV}^{3/2}$$

7. **Substitute and Calculate N:**

   - Convert Planck's constant h to $\mathrm{J}\backslash\mathrm{cdotps}$:

$$h^3 = (6.626 \times 10^{-34}\ \mathrm{J}\backslash\mathrm{cdotps})^3 = 2.93 \times 10^{-101}\ \mathrm{J}^3\backslash\mathrm{cdotps}^3$$

7. **Substitute and Calculate N:**

- Convert Planck's constant h to $\mathrm{J}\backslash\mathrm{cdotps}$:

$$h^3 = (6.626 \times 10^{-34}\ \mathrm{J}\backslash\mathrm{cdotps})^3 = 2.93 \times 10^{-101}\ \mathrm{J}^3\backslash\mathrm{cdotps}^3$$

- Effective mass factor:

$$(2m_n^*)^{3/2} = (2 \times 1.08 \times 9.11 \times 10^{-31})^{3/2} \approx 2.71 \times 10^{-45}\ \mathrm{kg}^{3/2}$$

- Combine these to find $\frac{(2m_n^*)^{3/2}}{h^3}$:

$$\frac{(2m_n^*)^{3/2}}{h^3} = \frac{2.71 \times 10^{-45}}{2.93 \times 10^{-101}} \approx 9.25 \times 10^{55}\ \mathrm{J}^{-3}\backslash\mathrm{cdotps}^{-3}$$

- Combine with the integral result:

$$N = 9.25 \times 10^{55} \times 0.00034 = 3.15 \times 10^{52}\ \mathrm{states/cm^3}$$

Conclusion:

The number of quantum states in Silicon between E_c and $E_c + 0.05\ \mathrm{eV}$ at $T = 300\ \mathrm{K}$ is approximately $3.15 \times 10^{52}\ \mathrm{states/cm^3}$.

Problem:

Determine the number of quantum states N_c in the conduction band and N_v in the valence band of Silicon between given energy levels.

Given:

- Effective mass of electrons in the conduction band: $m_n^* = 1.08m_0$
- Effective mass of holes in the valence band: $m_p^* = 0.81m_0$
- Planck's constant: $h = 6.626 \times 10^{-34}$ J.s
- Thermal energy: $kT = 0.02585$ eV
- Energy range for conduction band: from E_c to $E_c + 0.05$ eV
- Energy range for valence band: from $E_v - 0.05$ eV to E_v

Solution:

1. **Convert Thermal Energy kT to Joules:**

$$kT = 0.02585 \text{ eV} \times 1.602 \times 10^{-19} \text{ J/eV} = 4.14 \times 10^{-21} \text{ J}$$

2. **Density of States Function in the Conduction Band $g_c(E)$:**

The density of states $g_c(E)$ for conduction band electrons is given by:

$$g_c(E \geq E_c) = \frac{4\pi(2m_n^*)^{3/2}}{h^3}\sqrt{E - E_c}$$

3. **Density of States Function in the Valence Band $g_v(E)$:**

The density of states $g_v(E)$ for valence band holes is given by:

$$g_v(E \leq E_v) = \frac{4\pi(2m_p^*)^{3/2}}{h^3}\sqrt{E_v - E}$$

4. **Integrals to Find Number of States N_c and N_v:**
   - For the conduction band (from E_c to $E_c + 0.05\ \text{eV}$):

$$N_c = \int_{E_c}^{E_c+0.05\ \text{eV}} g_c(E)\, dE$$

   - For the valence band (from $E_v - 0.05\ \text{eV}$ to E_v):

$$N_v = \int_{E_v-0.05\ \text{eV}}^{E_v} g_v(E)\, dE$$

5. **Substitute the Expression for $g_c(E)$ and $g_v(E)$:**
   - For the conduction band:

$$N_c = \int_{E_c}^{E_c+0.05\ \text{eV}} \frac{4\pi(2m_n^*)^{3/2}}{h^3}\sqrt{E - E_c}\, dE$$

   - For the valence band:

$$N_v = \int_{E_v-0.05\ \text{eV}}^{E_v} \frac{4\pi(2m_p^*)^{3/2}}{h^3}\sqrt{E_v - E}\, dE$$

6. **Change of Variables and Evaluate the Integrals:**
   - **Conduction Band:**

Let $x = E - E_c$, so $dE = dx$. The limits become 0 to 0.05 eV:

$$N_c = \frac{4\pi(2m_n^*)^{3/2}}{h^3}\int_0^{0.05\ \text{eV}} \sqrt{x}\, dx$$

Evaluate the integral:

$$\int_0^{0.05\ \text{eV}} \sqrt{x}\, dx = \frac{2}{3}x^{3/2}\Big|_0^{0.05}$$

$$(0.05)^{3/2} = 0.00052$$

$$\int_0^{0.05\ \text{eV}} \sqrt{x}\, dx \approx \frac{2}{3} \times 0.00052 = 0.00034$$

- **Valence Band:**

  Let $x = E_v - E$, so $dE = -dx$. The limits become 0 to 0.05 eV:

$$N_v = \frac{4\pi(2m_p^*)^{3/2}}{h^3} \int_0^{0.05\ \mathrm{eV}} \sqrt{x}\, dx$$

  The integral is the same as for the conduction band:

$$\int_0^{0.05\ \mathrm{eV}} \sqrt{x}\, dx = 0.00034$$

Substitute and Calculate N_c and N_v:

- Convert Planck's constant h to $\mathrm{J.s}$:

$$h^3 = (6.626 \times 10^{-34}\ \mathrm{J.s})^3 = 2.93 \times 10^{-101}\ \mathrm{J^3.s^3}$$

- Effective mass factor for conduction band:

$$(2m_n^*)^{3/2} = (2 \times 1.08 \times 9.11 \times 10^{-31})^{3/2} \approx 2.71 \times 10^{-45}\ \mathrm{kg}^{3/2}$$

- Effective mass factor for valence band:

$$(2m_p^*)^{3/2} = (2 \times 0.81 \times 9.11 \times 10^{-31})^{3/2} \approx 2.04 \times 10^{-45}\ \mathrm{kg}^{3/2}$$

- Combine to find $\frac{(2m_n^*)^{3/2}}{h^3}$ and $\frac{(2m_p^*)^{3/2}}{h^3}$:

$$\frac{(2m_n^*)^{3/2}}{h^3} = \frac{2.71 \times 10^{-45}}{2.93 \times 10^{-101}} \approx 9.25 \times 10^{55}\ \mathrm{J^{-3}.s^{-3}}$$

$$\frac{(2m_p^*)^{3/2}}{h^3} = \frac{2.04 \times 10^{-45}}{2.93 \times 10^{-101}} \approx 6.95 \times 10^{55}\ \mathrm{J^{-3}.s^{-3}}$$

- Combine with the integral results:

$$N_c = 9.25 \times 10^{55} \times 0.00034 = 3.15 \times 10^{52}\ \mathrm{states/cm^3}$$

$$N_v = 6.95 \times 10^{55} \times 0.00034 = 2.36 \times 10^{52}\ \mathrm{states/cm^3}$$

Conclusion:

- The number of quantum states in the conduction band of Silicon between E_c and $E_c + 0.05\ \mathrm{eV}$ is approximately $3.15 \times 10^{52}\ \mathrm{states/cm^3}$.
- The number of quantum states in the valence band of Silicon between $E_v - 0.05\ \mathrm{eV}$ and E_v is approximately $2.36 \times 10^{52}\ \mathrm{states/cm^3}$.

This revised solution uses the period (.) for multiplication and provides clear calculations for both the conduction and valence bands.

Problem Statement

In a silicon semiconductor, determine the number of available quantum states in the conduction band and valence band within specific energy ranges.

Given:

- **Effective Mass of Electrons (Conduction Band):** $m_n^* = 1.08m_0$, where m_0 is the free electron mass.
- **Effective Mass of Holes (Valence Band):** $m_p^* = 0.81m_0$.
- **Planck's Constant:** $h = 6.626 \times 10^{-34}$ J.s.
- **Thermal Energy at Room Temperature (300 K):** $kT = 0.02585$ eV.
- **Energy Range for Conduction Band:** From E_c to $E_c + 0.05$ eV.
- **Energy Range for Valence Band:** From $E_v - 0.05$ eV to E_v.

Task:

1. Calculate the number of quantum states N_c in the conduction band between E_c and $E_c + 0.05$ eV.
2. Calculate the number of quantum states N_v in the valence band between $E_v - 0.05$ eV and E_v.

Solution

1. Number of Quantum States in the Conduction Band (N_c)

To find N_c, integrate the density of states function $g_c(E)$ over the given energy range. The density of states for the conduction band is:

$$g_c(E \geq E_c) = \frac{4\pi(2m_n^*)^{3/2}}{h^3}\sqrt{E - E_c}$$

Integrate this from E_c to $E_c + 0.05\ \text{eV}$:

$$N_c = \int_{E_c}^{E_c+0.05\ \text{eV}} g_c(E)\, dE$$

Substitute $g_c(E)$:

$$N_c = \int_{E_c}^{E_c+0.05\ \text{eV}} \frac{4\pi(2m_n^*)^{3/2}}{h^3}\sqrt{E - E_c}\, dE$$

Change the variable $x = E - E_c$, so $dE = dx$:

$$N_c = \frac{4\pi(2m_n^*)^{3/2}}{h^3}\int_{0}^{0.05\ \text{eV}} \sqrt{x}\, dx$$

Evaluate the integral:

$$\int_0^{0.05\text{ eV}} \sqrt{x}\, dx = \frac{2}{3} x^{3/2} \Big|_0^{0.05} = \frac{2}{3} \times (0.05)^{3/2}$$

$$(0.05)^{3/2} \approx 0.00052$$

$$\frac{2}{3} \times 0.00052 = 0.00034\text{ eV}$$

Thus:

$$N_c = \frac{4\pi (2m_n^*)^{3/2}}{h^3} \times 0.00034$$

2. Number of Quantum States in the Valence Band (N_v)

To find N_v, integrate the density of states function $g_v(E)$ over the specified energy range. The density of states for the valence band is:

$$g_v(E \leq E_v) = \frac{4\pi (2m_p^*)^{3/2}}{h^3} \sqrt{E_v - E}$$

Integrate this from $E_v - 0.05\text{ eV}$ to E_v:

$$N_v = \int_{E_v - 0.05\text{ eV}}^{E_v} g_v(E)\, dE$$

Substitute $g_v(E)$:

$$N_v = \int_{E_v - 0.05\text{ eV}}^{E_v} \frac{4\pi(2m_p^*)^{3/2}}{h^3} \sqrt{E_v - E}\, dE$$

Change the variable $x = E_v - E$, so $dE = -dx$:

$$N_v = \frac{4\pi(2m_p^*)^{3/2}}{h^3} \int_0^{0.05\text{ eV}} \sqrt{x}\, dx$$

The integral result is the same:

$$\int_0^{0.05\text{ eV}} \sqrt{x}\, dx = 0.00034\text{ eV}$$

Thus:

$$N_v = \frac{4\pi(2m_p^*)^{3/2}}{h^3} \times 0.00034$$

3. Substitute Values and Compute

Convert Planck's constant h to $\mathrm{J.s}$:

$$h^3 = (6.626 \times 10^{-34}\ \mathrm{J.s})^3 = 2.93 \times 10^{-101}\ \mathrm{J^3.s^3}$$

Effective mass factor for conduction band:

$$(2m_n^*)^{3/2} = (2 \times 1.08 \times 9.11 \times 10^{-31})^{3/2} \approx 2.71 \times 10^{-45}\ \mathrm{kg}^{3/2}$$

Effective mass factor for valence band:

$$(2m_p^*)^{3/2} = (2 \times 0.81 \times 9.11 \times 10^{-31})^{3/2} \approx 2.04 \times 10^{-45}\ \mathrm{kg}^{3/2}$$

Calculate $\frac{(2m_n^*)^{3/2}}{h^3}$ and $\frac{(2m_p^*)^{3/2}}{h^3}$:

$$\frac{(2m_n^*)^{3/2}}{h^3} = \frac{2.71 \times 10^{-45}}{2.93 \times 10^{-101}} \approx 9.25 \times 10^{55}\ \mathrm{J^{-3}.s^{-3}}$$

$$\frac{(2m_p^*)^{3/2}}{h^3} = \frac{2.04 \times 10^{-45}}{2.93 \times 10^{-101}} \approx 6.95 \times 10^{55}\ \mathrm{J^{-3}.s^{-3}}$$

Finally, the number of quantum states:

$$N_c = 9.25 \times 10^{55} \times 0.00034 = 3.15 \times 10^{52} \text{ states/cm}^3$$

$$N_v = 6.95 \times 10^{55} \times 0.00034 = 2.36 \times 10^{52} \text{ states/cm}^3$$

Conclusion

- The number of quantum states in the conduction band of Silicon between E_c and $E_c + 0.05 \text{ eV}$ is approximately $3.15 \times 10^{52} \text{ states/cm}^3$.
- The number of quantum states in the valence band of Silicon between $E_v - 0.05 \text{ eV}$ and E_v is approximately $2.36 \times 10^{52} \text{ states/cm}^3$.

Determine the number of quantum states N_c in the conduction band and N_v in the valence band of Silicon between given energy levels. Use different energy ranges for these calculations.

Given:

- Effective Mass of Electrons (Conduction Band): $m_n^* = 1.08m_0$
- Effective Mass of Holes (Valence Band): $m_p^* = 0.81m_0$
- Planck's Constant: $h = 6.626 \times 10^{-34}$ J.s
- Thermal Energy at Room Temperature (300 K): $kT = 0.02585$ eV

Example 1: Energy Range E_c to $E_c + 3kT$

1. Number of Quantum States in the Conduction Band (N_c)

Energy Range: From E_c to $E_c + 3kT$.

$$E_c + 3kT = E_c + 3 \times 0.02585 \text{ eV} = E_c + 0.07755 \text{ eV}$$

The density of states function $g_c(E)$ for the conduction band is:

$$g_c(E \geq E_c) = \frac{4\pi(2m_n^*)^{3/2}}{h^3}\sqrt{E - E_c}$$

To find N_c:

$$N_c = \int_{E_c}^{E_c+0.07755 \text{ eV}} \frac{4\pi(2m_n^*)^{3/2}}{h^3}\sqrt{E - E_c}\, dE$$

Substitute $x = E - E_c$, so $dE = dx$:

$$N_c = \frac{4\pi(2m_n^*)^{3/2}}{h^3}\int_0^{0.07755 \text{ eV}} \sqrt{x}\, dx$$

Evaluate the integral:

$$\int_0^{0.07755 \text{ eV}} \sqrt{x}\, dx = \frac{2}{3} x^{3/2} \Big|_0^{0.07755} = \frac{2}{3} \times (0.07755)^{3/2}$$

$$(0.07755)^{3/2} \approx 0.0021$$

$$\frac{2}{3} \times 0.0021 = 0.0014 \text{ eV}$$

Thus:

$$N_c = \frac{4\pi (2m_n^*)^{3/2}}{h^3} \times 0.0014$$

2. Number of Quantum States in the Valence Band (N_v)

Energy Range: From $E_v - 0.07755 \text{ eV}$ to E_v.

$$E_v - (E_v - 0.07755 \text{ eV}) = 0.07755 \text{ eV}$$

The density of states function $g_v(E)$ for the valence band is:

$$g_v(E \leq E_v) = \frac{4\pi(2m_p^*)^{3/2}}{h^3}\sqrt{E_v - E}$$

To find N_v:

$$N_v = \int_{E_v - 0.07755\ \mathrm{eV}}^{E_v} \frac{4\pi(2m_p^*)^{3/2}}{h^3}\sqrt{E_v - E}\,dE$$

Substitute $x = E_v - E$, so $dE = -dx$:

$$N_v = \frac{4\pi(2m_p^*)^{3/2}}{h^3}\int_0^{0.07755\ \mathrm{eV}} \sqrt{x}\,dx$$

The integral result is the same:

$$\int_0^{0.07755\ \mathrm{eV}} \sqrt{x}\,dx = 0.0014\,\mathrm{eV}$$

Thus:

$$N_v = \frac{4\pi(2m_p^*)^{3/2}}{h^3} \times 0.0014$$