Differential eq.
2nd Lecture
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2nd Stage
physical Dept
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# Methods of Solving First order, First Degree Differential Equations

In this section we shall discuss methods ~~for~~ of solving first order first degree diff. eq.

## (1)- Differential equations with variables separable

A first order first degree diff. eq. is of the form

$$\frac{dy}{dx} = F(x,y) \quad \text{--- (1)}$$

if $F(x,y)$ Can be expressed as a product $g(x)\,h(y)$, where $g(x)$ is function of $x$ and $h(y)$ is function of $y$, then the diff. eq (1) is said to be of variable separable type. The diff. eq (1) then has the form

$$\frac{dy}{dx} = g(x)\,h(y) \quad \text{--- (2)}$$

if $h(y) \neq 0$, separating the variabe, eq (2) can be rewritten as

$$\frac{1}{h(y)}\,dy = g(x)\,dx \quad \text{--- (3)}$$

integrating both side of eq. (3), we get

$$\int \frac{1}{h(y)} = \int g(x)\,dx \quad \text{--- (4)}$$

we get

$$L.H.S = 9e^{-3x} + (-3e^{-3x}) - 6.e^{-3x}$$
$$= 9e^{-3x} - 9e^{-3x} = 0 = R.H.S.$$

Therefore, the given function is a solution of the given diff. eq.

EX (3): Verify that the function $y = a\cos x + b\sin x$ where $a, b \in \mathbb{R}$ is a solution of the diff. eq

$$\frac{d^2y}{dx^2} + y = 0 \quad \text{--- } (*)$$

Solution

The given function is

$$y = a\cos x + b\sin x \quad \text{--- } (1)$$

Differetiating both sides of eq (1) with respect to $x$

$$\frac{dy}{dx} = -a\sin x + b\cos x \quad \text{--- } (2)$$

$$\frac{d^2y}{dx^2} = -a\cos x - b\sin x \quad \text{--- } (3)$$

Substituting the values $y$ (eq (1)) and $\frac{d^2y}{dx^2}$ (eq(3)). in given diff. eq. (*), we get

$$L.H.S = (-a\cos x - b\sin x) + (a\cos x + b\sin x)$$
$$= 0 = R.H.S.$$

$$\boxed{\text{How 2 : EXE 9.2}}$$

Thus, eq(4) providedes the solutions of given diff. eq. in the form

$$H(y) = G(x) + C$$

Here, $H(y)$ and $G(x)$ are the anti derivatives of $\frac{1}{h(y)}$ and $g(x)$ respectively and $C$ is the arbitrary constant.

Example 4: The general solution of diff. eq

$$\frac{dy}{dx} = \frac{x+1}{2-y}, \quad (y \neq 2)$$

Solution: we have

$$\frac{dy}{dx} = \frac{x+1}{2-y} \quad \dots (1)$$

separating the variable in eq(1), we get

$$(2-y)\,dy = (x+1)\,dx \quad \dots (2)$$

integrating both sides of eq(2), we get

$$\int (2-y)\,dy = \int (x+1)\,dx$$

$$\left[\, 2y - \frac{y^2}{2} = \frac{x^2}{2} + x + C_1 \,\right] * 2$$

$$\Rightarrow x^2 + y^2 + 2x - 4y + 2C_1 = 0$$

$$\Rightarrow x^2 + y^2 + 2x - 4y + C = 0, \quad \text{where } C = 2C_1$$

which is the general solution of eq(1)

**Example (5):** Find the general solution of the diff. eq. $\dfrac{dy}{dx} = \dfrac{1+y^2}{1+x^2}$

**Sol**

$\because$ $1+y^2 \neq 0$, therefor separating the variable the given diff. eq. can be written as

$$\frac{dy}{1+y^2} = \frac{dx}{1+x^2} \quad \text{--- (1)}$$

integrating both sides of eq (1), we get

$$\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$$

$$\Rightarrow \tan^{-1} y = \tan^{-1} x + c$$

which is the general solution of eq (1)

**Example (6)** Find the particular solution of the diff. eq. $\dfrac{dy}{dx} = -4xy^2$ given $y =$ , when $x = 0$

**Sol** if $y \neq 0$, the given diff. eq. can be written as

$$\Rightarrow \left[ -4xy^2\, dx = dy \right] \div y^2$$

$$\Rightarrow -4x\, dx = \frac{dy}{y^2} \quad \text{--- (1)}$$

Integrating both sides of eq (1), we get

$$\int \frac{dy}{y^2} = -4 \int x\, dx$$

$$\Rightarrow \int y^{-2}\,dy = -4\int x\,dx$$

$$\Rightarrow \frac{y^{-1}}{-1} = -4\frac{x^2}{2} + c$$

$$\Rightarrow \left(-\frac{1}{y} = -2x^2 + c\right) * -1$$

$$\Rightarrow \left[+1 = (2x^2 - c)y\right]$$

$$\Rightarrow y = \frac{1}{2x^2 - c} \quad \text{.... (2)}$$

Substituting $y=1$ and $x=0$ in eq (2), we get

$$y = \frac{1}{2(0) - c} \Rightarrow -c = 1 \Rightarrow \boxed{c = -1}$$

Now substituting the value of $c$ in eq (2) we get the particular solution of the given diff. eq as

$$y = \frac{1}{2x^2+1}.$$

$\boxed{\text{H.W 3:}}$ EXE 9.3