Differential equation
1st. Lecture
2/10/2024

Dr. mayadah. Khalil
2nd. stage
Physics. Dept.

Differential Equations

Def(1): An equation that contains derivative of unknown function is called differential equation.

Def(2): A differential equation (diff. eq.) always involves the derivative of one variable with respect to another. The former is called a dependent variable and the Latter an independent variable

dependent variable

$$\frac{dy}{dx} = g(x) \quad , \quad \text{where } y = f(x) ,$$

independent variable

Def(3): A diff. eq. involving only derivative with respect to one independent variable is called ordinary differential equation (ODE). otherewis it is called a ~~par~~ partial differential equations (PDE), which involving derivative ~~of independent variable~~ with respect more than one independent variable, for example:

(1) $2\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 0$ ODE

(2) $(3x+2y)^2 y' = 1$ ODE

(3) $(1+2y'^2)^{3/2} = 8y''$ ODE

(4) $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2z$ PDE

(5) $\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial z^2} = 0$ PDE

(6) $\left(\frac{\partial^2 z}{\partial x^2}\right)\left(\frac{\partial^2 z}{\partial y^2}\right) - \left(\frac{\partial^2 z}{\partial x \partial y}\right)^2 = 0$ PDE

note:- we using the following notations for derivatives:

① $\frac{dy}{dx} = y'$, ② $\frac{d^2y}{dx^2} = y''$, ③ $\frac{d^3y}{dx^3} = y'''$

④ for $\frac{d^ny}{dx^n} = y^n$

$\boxed{\text{~~Def (4): order of diff. eq. is the order of highest order derivative occurring in the differential equation~~}}$

Def (4): Ordere of differential equation:

order of a diff. eq. is defined as the order of The hights order derivative of dependent variable with respect to the independent variable involving in the given diff. eq.

(رتبة المعادلة هي رتبة اعلى مشتقة تظهر في المعادلة)

for example:

$$\frac{dy}{dx} = e^x \qquad \text{first order}$$

$$\frac{d^2y}{dx^2} + y = 0 \qquad \text{second order}$$

$$\left(\frac{d^3y}{dx^3}\right) + x^2\left(\frac{d^2y}{dx^2}\right)^3 = 0 \qquad \text{Third order}$$

Def (5): Degree of a differential equation (when define) is the highest power (postive integral only) of the highest order derivative

for example:

① $\left(\frac{d^3y}{dx^3}\right) + 2\left(\frac{d^2y}{dx^2}\right)^2 - \frac{dy}{dx} + y = 0$ degree one

② $\left(\frac{dy}{dx}\right)^2 + \frac{dy}{dx} - \sin^2 y = 0$ degree two

③ $\frac{dy}{dx} + \sin\left(\frac{dy}{dx}\right) = 0$ is not defen

• notation • Degree of a differential equation is define if it is a polynomial equation in its derivatives

• درجة المعادلة التفاضلية التي تكون جبرية في مشتقاتها هي الدرجة الجبرية للمشتقة ذات اعلى رتبة تظهر في المعادلة

• المعادلة التفاضلية غير جبرية في مشتقاتها ليس لها درجة

مثلا
$$\sin\left(\frac{dy}{dx}\right) = \frac{dy}{dx} + x + 3$$
ليس لها درجة

Example (1): Find the order and degree, if defined of each of the following differential equations.

(i) $\frac{dy}{dx} - \cos x$: first order, degree one

(ii) $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$:

∴ second order, degree one

(iii) $y''' + y^2 + e^{y'} = 0$:

third order.

since the diff eq. is not a polynomial in its derivative and so its degree is not define

بأن المعادلة التفاضلية ليست جبرية في مشتقاتها $(e^{y'})$ اذن الدرجة غير معرفة

$\boxed{p.3}$ H.w : $\boxed{\text{Exercise } 9.1}$

General and particular solution of a Differential Equation

الحل العام والحل الخاص للمعادلة التفاضلية.

Def (6): A function which satisfies the given diff. eq. is called its solution.

Def (7) The solution which contains as many arbitrary constants as the order of diff. eq. is called a general solution.

Def (8) The solution obtain from the general solution by giving particular values to the arbitrary constants is called a particular solution of the differential equation

Example 2: Verify that the function $y = e^{-3x}$ is a solution of the diff. eq.

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0 \quad \text{——} (*)$$

solution: Given function is $y = e^{-3x}$ Differentiating both sides of eq with respect to x, we get

$$\frac{dy}{dx} = -3e^{-3x} \quad \text{...}(1)$$

Now, differentiating (1) with respect to x, we have

$$\frac{d^2y}{dx^2} = 9e^{-3x} \quad \text{...}(2)$$

Substituting the values of $\frac{dy}{dx}$ (eq(1)) and $\frac{d^2y}{dx^2}$ (eq.2) and y in eq (*)

we get

$$L.H.S = 9e^{-3x} + (-3e^{-3x}) - 6 \cdot e^{-3x}$$
$$= 9e^{-3x} - 9e^{-3x} = 0 = R.H.S.$$

Therefore, the given function is a solution of the given diff. eq.

EX (3): Verify that the function $y = a\cos x + b\sin x$ where $a, b \in \mathbb{R}$ is a solution of the diff. eq.

$$\frac{d^2y}{dx^2} + y = 0 \quad \text{---} (*)$$

Solution

The given function is

$$y = a\cos x + b\sin x \quad \text{---} ①$$

Differetiating both sides of eq (1) with respect to x

$$\frac{dy}{dx} = -a\sin x + b\cos x \quad \text{---} ②$$

$$\frac{d^2y}{dx^2} = -a\cos x - b\sin x \quad \text{---} ③$$

substituting the values y (eq (1)) and $\frac{d^2y}{dx^2}$ (eq (3)). in given diff. eq. (*), we get

$$L.H.S = (-a\cos x - b\sin x) + (a\cos x + b\sin x)$$
$$= 0 = R.H.S.$$

$\boxed{\text{How 2 : EXE 9.2}}$