

EXE 9.3

For each of the diff. eq in EXE 1 to 10
find The general solution;

$$\textcircled{1} \frac{dy}{dx} = \frac{1 - \cos x}{1 + \cos x} \Rightarrow dy = \left(\frac{1 - \cos x}{1 + \cos x} \right) dx$$

$$dy = \frac{1 - \cos x}{1 + \cos x} \cdot \frac{1 - \cos x}{1 - \cos x} dx$$

$$dy = \frac{1 - 2\cos x + \cos^2 x}{1 - \cos^2 x} dx \quad \boxed{1 - \cos^2 x = \sin^2 x}$$

$$dy = \frac{1 - 2\cos x + \cos^2 x}{\sin^2 x} dx$$

$$dy = \frac{\sin^2 x - 2\cos x}{\sin^2 x} dx$$

$$dy = \left(1 - 2 \frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} \right) dx$$

$$dy = (1 - 2 \cot x \csc x) dx \quad \begin{array}{l} \text{integrating both} \\ \text{side we get} \end{array}$$

$$\int dy = \int dx - 2 \int \cot x \csc x dx$$

P.1

$$y = x^2 - 2(1 - \csc x) + C$$

$$\Rightarrow x^2 + 2 \csc x - y + C$$

is general solution for given diff. eq

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$$\textcircled{2} \quad \frac{dy}{dx} = \sqrt{4-y^2} \Rightarrow dy = \sqrt{4-y^2} dx$$

$$\Rightarrow \frac{1}{\sqrt{4-y^2}} dy = dx \Rightarrow (4-y^2)^{-\frac{1}{2}} dy = dx$$

$$\Rightarrow \int \frac{1}{\sqrt{2^2-y^2}} dy = \int dx$$

$$\int \frac{dy}{\sqrt{a^2+y^2}} = \sinh^{-1}\left(\frac{y}{a}\right) + C$$

$$\sinh^{-1}\left(\frac{y}{2}\right) = x + C$$

$$\Rightarrow x - \sinh^{-1}\left(\frac{y}{2}\right) + C$$

is general solution for given diff. eq.

$$3. \quad \frac{dy}{dx} + y = 1 \quad (y \neq 1)$$

$$\Rightarrow \frac{dy}{dx} = 1-y \Rightarrow dy = (1-y) dx$$

$$\left[\frac{dy}{(1-y)} = dx \right] \quad \text{integrating both sides}$$

$$-1 \int \frac{dy}{(1-y)} (-1) = \int dx$$

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$$- \ln(1-y) = x + C$$

$$\Rightarrow x + \ln(1-y) + C$$

is general solution for given diff. eq.

P.2

(ExE: 9.3)

④. $[\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0]$

divided both side on $\tan x \cdot \tan y$

we get

$$\frac{\sec^2 x}{\tan x} dx + \frac{\sec^2 y}{\tan y} dy = 0$$

integral both side

$$\int \frac{\sec^2 x}{\tan x} dx + \int \frac{\sec^2 y}{\tan y} dy = 0$$

$$\ln |\tan x| + \ln |\tan y| + c$$

is general solution of given diff. eq.

⑤ $e^x + e^{-x} dy - (e^x - e^{-x}) dx = 0$

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$$(e^x + e^{-x}) dy = (e^x - e^{-x}) dx$$

divided both side on $e^x + e^{-x}$

$$\frac{dy}{dx} = \left(\frac{e^x - e^{-x}}{e^x + e^{-x}} \right) dx$$

P-3

$$\therefore \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\Rightarrow dy = \tanh x dx \quad \text{integral both side}$$

$$\int dy = \int \tanh x dx$$

$$\int \tanh x dx = \operatorname{sech}^2 x$$

$$y = \operatorname{sech}^2 x + c$$

$$\operatorname{sech}^2 x - y + c$$

is general solution of given diff. eq

EXE 9.3

$$\textcircled{6} \frac{dy}{dx} = (1+x^2)(1+y^2)$$

$$[dy = (1+x^2)(1+y^2) dx] \div (1+y^2)$$

$$\frac{dy}{1+y^2} = (1+x^2) dx \quad \text{integral both side}$$

$$\int \frac{dy}{1+y^2} = \int (dx) + \int x^2 dx$$

$$\tan^{-1} y = x + \frac{x^3}{3} + C$$

$$\int \frac{dy}{1+y^2} = \tan^{-1} y + C$$

$$\frac{x^3}{3} + x - \tan^{-1} y + C$$

is general solution for given diff. eq.

$$\textcircled{7} [y \log y dx - x dy = 0] \div (y \log y) (x)$$

$$\int \frac{dx}{x} - \int \frac{dy}{y \log y} \Rightarrow \ln x - \ln(\ln y) + C$$

$$\textcircled{8} x^5 \frac{dy}{dx} = -y^5$$

$$\boxed{P.4} [x^5 dy = -y^5 dx] -x^5 y^5$$

$$-\frac{dy}{y^5} = \frac{dx}{x^5}$$

$$-\int y^{-5} dy = \int x^{-5} dx -$$

$$\Rightarrow \left(\frac{-y^{-4}}{4} \right) = \left(\frac{x^{-4}}{4} \right) + C$$

$$\frac{1}{4y^4} + \frac{x^4}{4} + C$$

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EXE 9.3

9. $\frac{dy}{dx} = \sin^{-1} x$

$dy = \sin^{-1} x \, dx$ integral both side

$\int dy = \int \sin^{-1} x \, dx$

$y = \frac{1}{\sqrt{1-x^2}} + C$

$\int \sin^{-1} x = \frac{1}{\sqrt{1-x^2}} + C$

$\Rightarrow \frac{1}{\sqrt{1-x^2}} = y + C$

10. $e^x \tan y \, dx + (1-e^x) \sec^2 y \, dy = 0$

divided by $(\tan y (1-e^x))$

$\frac{e^x}{1-e^x} \, dx + \frac{\sec^2 y}{\tan y} \, dy = 0$

integral both side

$\frac{1}{-1} \int \frac{e^x \cdot (-1)}{1-e^x} \, dx + \int \frac{\sec^2 y}{\tan y} \, dy$

$= -\ln(1-e^x) + \ln |\tan y| + C$

P.S

EXE 9.4

In each of the ExE 1-10, show that the given diff eq is homogenous and solve each of them.

1. $(x^2 + xy) dy = (x^2 + y^2) dx$

$$\frac{dy}{dx} = \frac{x^2 + xy}{x^2 + y^2}$$

$$\frac{x^2(1 + \frac{y}{x})}{x^2(1 + \frac{y^2}{x^2})} = \frac{1 + \frac{y}{x}}{1 + (\frac{y}{x})^2} \quad \text{--- (1)}$$

It is a diff. eq. of the form $\frac{dy}{dx} = F(x,y)$

$$\text{Here } F(x,y) = \frac{1 + \frac{y}{x}}{1 + (\frac{y}{x})^2} \quad \text{--- (2)}$$

Replacing x by λx and y by λy , we get

$$F(\lambda x, \lambda y) = \frac{1 + \frac{\lambda y}{\lambda x}}{1 + \frac{\lambda y^2}{\lambda^2 x^2}} = \lambda^0 [F(x,y)]$$

Thus $F(\lambda x, \lambda y)$ is homogenous function of degree zero.

To solve it we make the substitution

$$y = vx \quad \text{--- (2)}$$

$$v = \frac{y}{x} \quad \text{--- (3)}$$

Differentiating eq (2) with respect to x

$$\frac{dy}{dx} = v + x \frac{dv}{dx} \quad \text{--- (4)}$$

Substituting values of eq (2) and eq (3) and eq. (4) in eq. (1) we get

P.6

$$v + x \frac{dv}{dx} = \frac{1+v}{1+v^2}$$

$$x \frac{dv}{dx} = \frac{1+v}{1+v^2} - v$$

$$x \frac{dv}{dx} = \frac{1+v^2 - v - v^2}{1+v^2}$$

~~$$x \frac{dv}{dx} = \frac{1-v}{1+v}$$~~

$$x(1+v) dv = (1-v) dx$$

$$\int \frac{(1+v)}{(1-v)} dv = \int \frac{dx}{x}$$

$$\int \frac{(1+v)}{(1-v)} \cdot \frac{(1-v)}{(1-v)} dv = \int \frac{dx}{x}$$

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$$\int \left(\frac{1-v - v - v^2}{1-v^2} \right) dv = \ln x + c$$

$$\int \left(\frac{1-2v-v^2}{1-v^2} \right) dv = \ln x + c$$

$$\int \frac{-2v}{1-v^2} + 1 \cdot dv = \ln x + c$$

P. 7

$$\text{Log}(1-v^2) + v = \text{Log} x + c$$

$$\text{Log} x - \text{Log}(1-v^2) + v + c \quad \dots \quad (5)$$

replacing v by $\frac{y}{x}$ in eq(5) we get

$$\log x - \log \left(1 - \frac{y^2}{x^2} \right) - \frac{y}{x} + c$$

$$\log x - \log \left(\frac{x^2 - y^2}{x^2} \right) - \frac{y}{x} + c$$

$$\log \left(\frac{x}{x^2 - y^2} \right) - \frac{y}{x} + c \Rightarrow \log \left(\frac{x^3}{x^2 - y^2} \right) - \frac{y}{x} + c$$

EXE: 9.4

$$(2) \frac{dy}{dx} = \frac{x+y}{x} \quad \rightarrow (1)$$

نفس الخطوات المذكورة للقرين (1) في (P.6) لاختيار التجانس homogenous function
يجب اعتبارها ككتابة الخطوات مرة

من نبدأ هنا من الخطوة الثانية وهي الفرضية (الخطوات المذكورة على الجانب)

$$\therefore \frac{dy}{dx} = \frac{x(1+\frac{y}{x})}{x} = \frac{dy}{dx} = 1 + \frac{y}{x} \quad (1)$$

To solve it we make the substitution

$$y = vx \quad (2)$$

$$v = \frac{y}{x} \quad (3)$$

Differentiating eq (2) with respect to x

$$\frac{dy}{dx} = v + x \frac{dv}{dx} \quad (4)$$

Substituting ^{value of} eq (2), eq (3) and eq (4) in eq (1) we
we get

$$v + x \frac{dv}{dx} = 1 + v$$

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$$x \frac{dv}{dx} = 1 + v - v \Rightarrow x \frac{dv}{dx} = 1$$

$$\Rightarrow [x dv = dx] \div x$$

$$\Rightarrow dv = \frac{dx}{x}$$

integrating both side

$$\int dv = \int \frac{dx}{x} \Rightarrow$$

P.85

$$v = \log x + C \Rightarrow \log x - v + C$$

replacing v by $\frac{y}{x}$ we get

$$\log x - \frac{y}{x} + C$$

is general solution for give the diff. eq.

$$(4) (x^2 - y^2) dx + 2xy dy = 0$$

$$\frac{dy}{dx} = \frac{x^2 - y^2}{2xy}$$

نختبر انهما متجانسا (يترك للطالب ان يثبتها بحسب خطواته اولى)

$$\frac{dy}{dx} \frac{x^2 (1 - \frac{y^2}{x^2})}{2x (\frac{y}{x^2})} = \frac{\frac{x^2}{x^2} - \frac{y^2}{x^2}}{\frac{2xy}{x^2}}$$

$$\frac{dy}{dx} = \frac{1 - (\frac{y}{x})^2}{2 \frac{y}{x}} \quad - (1)$$

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To solve it we make

$$y = vx \quad - (2)$$

$$v = \frac{y}{x} \quad - (3)$$

differential eq (2) with respect to x

$$\frac{dy}{dx} = v + x \frac{dv}{dx} \quad - (4)$$

Substituting values of eq (2), eq (3) and eq (4) in eq (1), we get

$$v + x \frac{dv}{dx} = \frac{1 - v^2}{2v}$$

$$x \frac{dv}{dx} = \frac{1 - v^2}{2v} - v \Rightarrow x \frac{dv}{dx} = \frac{1 - v^2 - 2v^2}{2v}$$

$$\left[x dv = \frac{3v^2}{2v} dx \right] \div (x) \left(\frac{1 - 3v^2}{2v} \right)$$

$$-\frac{1}{3} \int \frac{2v}{1 - 3v^2} dv = \int \frac{1}{x} dx \quad (P.9)$$

$$-\frac{1}{3} \log(1 - 3v^2) = \log x + c$$

$$\log x + \log (1-3v^2)^{\frac{1}{3}} + C$$

$$\log (x \cdot \sqrt[3]{1-3v^2}) + C$$

replacing $v = \frac{y}{x}$ we get

$$\log (x \sqrt[3]{1-3(\frac{y}{x})^2}) + C$$

الخطوات الاربعة لاختيار $v = \frac{y}{x}$ وانه $v = \frac{y}{x}$ $\Rightarrow y = vx$ $\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$

$$\textcircled{2} \quad x dy - y dx = \sqrt{x^2 + y^2} dx$$

$$\rightarrow x dy = (\sqrt{x^2 + y^2} - y) dx$$

$$\frac{dy}{dx} = \frac{\sqrt{x^2 + y^2} - y}{x}$$

P.10

نقسم البسط والقام على x و نأخذ $\frac{y}{x}$ كن v كان $x^2 \sqrt{x^2}$

$$\frac{dy}{dx} = \frac{\sqrt{\frac{x^2}{x^2} + \frac{y^2}{x^2}} - \frac{y}{x}}{1} \Rightarrow \frac{dy}{dx} = \sqrt{1 + \frac{y^2}{x^2}} - \frac{y}{x} \quad \textcircled{1}$$

To make solve, if we make

$$y = vx \quad \textcircled{2}$$

$$v = \frac{y}{x} \quad \textcircled{3}$$

differential both side for eq 2 we get

$$\frac{dy}{dx} = v + x \frac{dv}{dx} \quad \textcircled{4}$$

Substitution values ~~for~~ of eq (2), eq (3) and eq (4) in eq (1), we get

EXE 9.4

$$v + x \frac{dv}{dx} = \sqrt{1+v^2} - v$$

$$x \frac{dv}{dx} = \sqrt{1+v^2} - v + v$$

$$[x dv = (\sqrt{1+v^2}) dx] \div x \sqrt{1+v^2}$$

$$\frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}$$

integration for both side

$$\int \frac{dv}{\sqrt{1+v^2}} = \int \frac{dx}{x}$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c$$

$$\sin^{-1} v = \log x + c$$

$$\Rightarrow \log x - \sin^{-1} v + c$$

replacing v by $\frac{y}{x}$, we get

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$$\log x - \sin^{-1} \left(\frac{y}{x}\right) + c$$

which is general solution for given diff. eq

$$\textcircled{7} \{x \cos \frac{y}{x} + y \sin(\frac{y}{x})\} y dx$$

$$= \left\{ y \sin \left(\frac{y}{x}\right) - x \cos \left(\frac{y}{x}\right) \right\} x dy$$

P.11

$$\text{Sol } xy \cos \left(\frac{y}{x}\right) + y^2 \sin \left(\frac{y}{x}\right) dx = xy \sin \left(\frac{y}{x}\right) - x^2 \cos \left(\frac{y}{x}\right) dy$$

$$\frac{dy}{dx} = \frac{xy \cos \left(\frac{y}{x}\right) + y^2 \sin \left(\frac{y}{x}\right)}{xy \sin \left(\frac{y}{x}\right) - x^2 \cos \left(\frac{y}{x}\right)}$$

تقسيم البنية المقام به x^2

$$\frac{dy}{dx} = \frac{\frac{xy}{x^2} \cos\left(\frac{y}{x}\right) + \frac{y^2}{x^2} \sin\left(\frac{y}{x}\right)}{\frac{xy}{x^2} \sin\left(\frac{y}{x}\right) - \frac{x^2}{x^2} \cos\left(\frac{y}{x}\right)}$$

$$\frac{dy}{dx} = \frac{\frac{y}{x} \cos\left(\frac{y}{x}\right) + \left(\frac{y}{x}\right)^2 \sin\left(\frac{y}{x}\right)}{\frac{y}{x} \sin\left(\frac{y}{x}\right) - \cos\left(\frac{y}{x}\right)} \quad \text{--- (1)}$$

نريد ان نتحقق ان المعادلة متجانسة (homogenous func)

To solve, it make $y = vx$ --- (2)

$$v = \frac{y}{x} \quad \text{--- (3)}$$

Differentiate both side of eq (2) we get

$$\frac{dy}{dx} = v + x \frac{dv}{dx} \quad \text{--- (4)}$$

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Substitution values of eq (2), eq (3) and eq (4) in eq (1), we get

$$v + x \frac{dv}{dx} = \frac{v \cos v + v^2 \sin v}{v \sin v - \cos v}$$

P. 12

$$x \frac{dv}{dx} = \frac{v \cos v + v^2 \sin v}{v \sin v - \cos v} - v$$

$$x \frac{dv}{dx} = \frac{v \cos v + v^2 \sin v - v^2 \sin v + v \cos v}{v \sin v - \cos v}$$

$$x \frac{dv}{dx} = \frac{2v \cos v}{v \sin v - \cos v}$$

$$\left[x \, dv = 2 \frac{v \cos v}{v \sin v - \cos v} dx \right] \div x \frac{v \cos v}{v \sin v - \cos v}$$

EXE 9.4

$$\left(\frac{v \sin v - \cos v}{v \cos v} \right) dv = 2 \frac{dx}{x}$$

therefor

$$\int \frac{v \sin v - \cos v}{v \cos v} dv = 2 \int \frac{dx}{x}$$

$$\int \left(\frac{\cancel{v} \sin v}{\cancel{v} \cos v} - \frac{1}{v} \right) dv = 2 \int \frac{dx}{x}$$

$$\int \tan v \, dv - \int \frac{dv}{v} = 2 \int \frac{dx}{x}$$

$$\log |\sec v| - \log v = 2 \log x + \log |C_1|$$

$$\log \left| \frac{\sec v}{v} \right| = 2 \log |x^2| + \log |C_1|$$

$$\log \left| \frac{\sec v}{v} \right| - \log |x^2| = \log |C_1|$$

$$\frac{\log |a| - \log |b|}{= \log \left| \frac{a}{b} \right|}$$

$$n \log a = \log a^n$$

$$\left[\log \left| \frac{\sec v}{v x^2} \right| = \log |C_1| \right] \quad * e$$

$$\frac{\sec v}{v x^2} = \pm C_1$$

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Replacing v by $\frac{y}{x}$ in eq (5), we get

$$\frac{\sec \frac{y}{x}}{\frac{y}{x} x^2} = C \quad \text{where } C = \pm C_1$$

P-13

$$\Rightarrow \frac{\sec \left(\frac{y}{x} \right)}{yx} = C \Rightarrow \sec \frac{y}{x} = C xy$$

which is general solution of the give diff. eq.

ExE 9.4

(8) x dy/dx - y + x sin(y/x) = 0

x dy/dx = y - x sin(y/x)

dy/dx = [y - x sin(y/x)] / x => dy/dx = y/x - sin(y/x)

homogenous fun. ...

To solve it make it

y = vx (2)

v = y/x (3)

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Differentiate both side of eq (2) we get

v + x dv/dx (4)

Substitute values of eq (2), eq (3) and eq (4) in eq (1) we get

v + x dv/dx = v - sin(v)

x dv/dx = v - sin v

P.14

x dv/dx = - sin v

[x dv = - sin v dx] / (x) (- sin v)

int du/sin v = int dx/x

int sin^-1 v = int dx/x => v sin^-1 v + sqrt(1-v^2) = log x + c

=> log x - v sin^-1 v = sqrt(1-v^2) + c (5)

ExE 9.4

replacing u by $\frac{y}{x}$ in eq (3), we get

$$\log x - \frac{y}{x} \sin^{-1}\left(\frac{y}{x}\right) - \sqrt{1 - \left(\frac{y}{x}\right)^2} + C$$

which is general solution for given diff. eq.

(10) $(1 + e^{\frac{x}{y}}) dx + e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) dy = 0$

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EXE 9.5

② $\frac{dy}{dx} + 3y = e^{-2x}$

The given diff. eq. is of the form

$\frac{dy}{dx} + Py = Q$, where $P=3$ and $Q = e^{-2x}$

Therefore I.F = $e^{\int P dx} = e^{\int 3 dx} = e^{3x}$

$y * I.F = \int (Q * I.F) dx + c$

$y e^{3x} = \int e^{3x} \cdot e^{-2x} dx + c$

$y e^{3x} = \int e^x dx + c$

$[y e^{3x} = e^x + c] e^{-3x}$

$y = e^{-2x} + e^{-3x} c_1$

$y = e^{-2x} + c$, where $c = e^{-3x} c_1$

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③ $\frac{dy}{dx} + \frac{y}{x} = x^2$

Given diff. eq. is of the form

$\frac{dy}{dx} + Py = Q$, where $P = \frac{1}{x}$, $Q = x^2$

I.F = $e^{\int P dx} = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$

$y \cdot I.F = \int (Q * I.F) dx + c$

$xy = \int x^2 \cdot x + c \Rightarrow xy = \frac{x^4}{4} + c \Rightarrow$

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$$[xy = \frac{x^3}{3} + c] \div x$$

$$y = \frac{x^3}{3} + \frac{c}{x}$$

$$y = \frac{x^3}{3} + C_1, \text{ where } C_1 = \frac{c}{x}$$

$$\textcircled{4} \frac{dy}{dx} + \sec x y = \tan x$$

Given diff. eq. of form

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$$\frac{dy}{dx} + Py = Q \text{ where } P = \sec x, Q = \tan x$$

$$I.F = e^{\int P dx} = e^{\int \sec x dx} = e^{\ln |\sec x + \tan x|} = \sec x + \tan x$$

$$y * I.F = \int (Q * I.F) dx + c$$

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$$y (\sec x + \tan x) = \int \tan x (\tan x + \sec x) dx + c$$

$\tan^2 x = 1 + \sec^2 x$
 $\int \tan^2 x = \int dx + \int \sec^2 x$

$$= \int (\tan^2 x + \tan x \sec x) dx + c$$

$$= \int \tan^2 x + \int \tan x \sec x dx + c$$

$$y (\sec x + \tan x) = \tan x - x + \sec x + c$$

$$y = \frac{\tan x - x + \sec x + c}{\sec x + \tan x} = \frac{1}{\tan x} + \frac{1}{\sec x} - \frac{x}{\sec x + \tan x} + c$$

$$\textcircled{5} \cos^2 x \frac{dy}{dx} + y = \tan x$$

$$\frac{dy}{dx} + \frac{y}{\cos^2 x} = \frac{\tan x}{\cos^2 x}$$

Given this eq of form

$$\frac{dy}{dx} + Py = Q \quad \text{where } P = \frac{1}{\cos^2 x}$$

$$\begin{aligned} \text{I.F.} &= e^{\int P dx} = e^{\int \sec^2 x dx} \\ &= e^{\tan x} \end{aligned} \quad \text{where } Q = \frac{\tan x}{\cos^2 x}$$

$$y * \text{I.F.} = \int (Q * \text{I.F.}) dx + C$$

$$y \cdot e^{\tan x} = \int \left(\frac{\tan x}{\cos^2 x} * e^{\tan x} \right) dx$$

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